Armand Coudray

Curriculum Vitae

Experience

2024-2025 **Temporary Research and Teaching Agent (ATER)**, *University of Franche-Comté*, Besançon

2023-2024 **Temporary Research and Teaching Agent (ATER)**, *University of Burgundy*, Dijon

Higher education

2020-2024 **PhD thesis in mathematical physics : Asymptotic behaviour of zero rest-mass fields on radiative spacetimes**, *University of Western Brittany (UBO), LMBA*, Brest, Supervisor: Jean-Philippe Nicolas, defended on 6 May 2024.
My thesis deals with conformal analysis in two types of dynamical spacetimes that are solutions to the Einstein equations: Vaidya and Robinson-Trautman.

2019-2020 **Master 2 in Theoretical Physics**, *Aix-Marseille University (AMU)*, Marseille

2018-2019 **Master in Fundamental Physics and Applications**, *Sorbonne University (ex: UPMC)*, Paris

2015-2018 **Degree in Physics**, *University of Pau and Pays de l'Adour (UPPA)*, Anglet and Pau

Conferences

The Vaidya spacetime and its conformal compactification, *PhD seminar*, Dijon, February 2024

Conformal scattering in the Vaidya spacetime, *Lebesgue Doctoral Meeting 2023*, Nantes, April 2023

Behaviour of the event horizon in the radiative Robinson-Trautman spacetime, *Mathematical Physics working group*, LMBA, Brest, March 2023

Peeling-off property of scalar waves in Vaidya's spacetime, *Geometry seminar*, Institut Denis Poisson, Tours, March 2023

Peeling-off property of scalar waves in Vaidya's spacetime, *Mathematical Aspects of Black Hole Theory Workshop*, LUTH, Meudon, December 2022

Peeling-off property of scalar waves in Vaidya's spacetime, *Geometry seminar*, LMBA, Brest, December 2021

Working group

45 Rue des Justices – 25000 – Besançon, France
06 86 26 78 00 • armand.coudray@univ-fcomte.fr
https://acoudray.perso.math.cnrs.fr/

BMS group and BMS formalism, IMB, Dijon, 2023-2024
We are following the lecture series given by Abhay Ashtekar at the Institute for Gravitation and the Cosmos at Penn State. The objective of this working group is to comprehend the algebra of the BMS group and to understand the physical significance of the mathematical structure at infinity.

Invitation

March 2022-June 2022 **Mathematical Institute, Oxford, England**, *3 months stay on a invitation by Lionel Mason*, December 2022

Teaching

Functions Series, 30h, *Undergraduates*, UFC, Besançon, 2024/2025

Mathematics for Economy and Management, 25h, *Undergraduates*, UFC, Besançon, 2024/2025

Applied Analysis : ODE, 30h, *Undergraduates students*, UFC, Besançon, 2024/2025

Normed Vector Spaces, 60h, *Undergraduates*, UFC, Besançon, 2024/2025

Mathematics for Economy : Algebra, 20h, *Undergraduates*, UB, Dijon, 2023/2024

Statistics and Data Analysis in Biology, 20h, *Undergraduates*, UB, Dijon, 2023/2024

Mathematics for physicists and engineers, 40h, *Undergraduates*, UB, Dijon, 2023/2024

Statistics, 55h, *Undergraduates*, UB, Dijon, 2023/2024

Mathematics for physicists, 30h, *Undergraduates*, UBO, Brest, 2022/2023

Analysis I, 30h, *Undergraduates in mathematics, 30h*, UBO, Brest, 2022/2023

Supervised a Graduate's level student in a research project., *UBO, Brest*, We are interested in the Schwarzschild metric and will derive the existence of two specific spherical orbits: the photon sphere and the innermost stable circular orbit (ISCO) for massive particles. The goal of the project was to introduce the student to General Relativity by utilising effective potential methods, 2022/2023

Mathematics for physicists, 30h, *Undergraduates*, UBO, Brest, 2021/2022

Publications

[1] Armand Coudray, *Conformal scattering of the wave equation in the Vaidya spacetime*, Submitted, (2024), *https://doi.org/10.48550/arXiv.2405.08659*.

[2] Armand Coudray, *Peeling-off behavior of wave equation in the Vaidya spacetime*, Journal of Hyperbolic Differential Equations, **20**, 02 (2023), *https://doi.org/10.1142/S021989162350011X*

[3] Armand Coudray and Jean-Philippe Nicolas, *Geometry of Vaidya spacetimes*, General Relativity and Gravitation, **53**, 73 (2021), *https://doi.org/10.1007/s10714-021-02839-7*.

[4] Armand Coudray, *Geometry of Vaidya-Bonnor spacetimes*, In Preparation.

[5] Armand Coudray and Grigalius Taujanskas, *Conformal scattering for Maxwell-Klein-Gordon fields in Minkowski spacetime*, In Preparation.

Research interest

My education is mainly as a physicist but I always had a strong interest in the mathematical structures underlying the physical theories and I was eager to explore this by doing a PhD in mathematical physics. My research so far is centred on radiative spacetimes and conformal methods in general relativity. Here are the details.

1. **Geometry of Vaidya's spacetimes [3]**. This research was developed during my Master 2 research project in Brest. We investigate the behaviour of null radial geodesics on Vaidya's spacetime in the case of an evaporating white hole. We study the dynamical behaviour of the past horizon and construct the second optical function that characterize null incoming radial geodesics. We devote special attention to the case of a complete evaporation and we show the existence of an asymptotic light-like singularity of the conformal curvature. The approach is not numerical but based on a rigorous analysis of the ordinary differential equations that govern the dynamics of null geodesics.

2. **Conformal analysis in Vaidya's spacetime [2], [1]**. We establish the peeling-off behaviour for scalar waves in Vaidya's spacetime for a class of data that is as large as the corresponding one in Minkowski spacetime [2]. We use conformal techniques and energy estimates to obtain the optimal classes of initial data ensuring a given regularity of the rescaled field at null infinity. We also devote particular attention to the peeling-off of the scalar waves in the past. Since the Vaidya metric is time-dependent, extending the results to the past is not trivial, contrary to the case of the Schwarzschild spacetime. We prove that the class of initial data ensuring a given regularity on the past null infinity is as large as the corresponding one in Minkowski spacetime. We also investigate conformal scattering for scalar waves in the case of partial evaporation and stabilization to a Schwarzschild black hole [1]. The construction of the scattering operator is as follows: firstly, we establish the existence of trace operators that associate the restriction of the rescaled field to future and past null boundaries (made of the union of the future/past horizon and the future/past null infinity) to the initial data of the field on a Cauchy hypersurface. These operators are initially constructed for smooth data with compact support. We then establish energy estimates in both directions between null boundaries and the Cauchy hypersurface. Since the metric we consider is the Schwarzschild metric near the future and past timelike infinities, we can use the results of M. Dafermos and I. Rodnianski (2008) to show that there is no energy of the rescaled field that goes to these singularities of the conformal boundary. This allows us to extend the trace operators by density to all data with finite energy without support constraints. Finally, by solving a Goursat problem (or a characteristic Cauchy problem) on the null boundary for smooth data with compact support, we prove that the trace operators act like isomorphisms between the hypersurface of initial data and the future/past boundaries.

3. **Geometry of radiative spacetimes [4]**. We investigate the geometry of Robinson-Trautman spacetime with a pure radiation field. We focus on studying a spacetime with a radiation density that is a flow of matter of zero rest-mass and is propagated along outgoing null radial geodesics.

In this case, the metric is expressed in terms of outgoing Eddington-Finkelstein coordinates (u, r, ω) and depends on two functions $m(u)$ and $P(u, \theta, \varphi)$. We assume that the metric is of an algebraically special type, specifically Petrov type D, and we know that solutions of this kind exist. We assume that m is a decreasing function on $\mathbb{R}$ with a finite limit as $u \to \pm\infty$. We also assume that P is a constant function on $-\infty, u_- \cup]u_+, +\infty[$ and varies only on $[u_-, u_+]$ with $u_- < u_+$. In other words, we consider a Vaidya white hole that transforms to another by non-spherical radiations on $[u_-, u_+]$.
The main difference with the Vaidya metric is that, in general, in pure radiative Robinson-Trautman spacetimes, the incoming principal null direction (PND) is not hypersurface forming; hence, it does not generate the horizon. It follows that the method developed in the Vaidya spacetime fails to provide information about the behaviour of the past horizon.
We now study the geometry of a model in which spherical symmetry is preserved: the Vaidya-Bonnor metric. This spacetime corresponds to a Vaidya spacetime in which the white hole has an electric charge q. This charge is allowed to depend on u, and this charge must be increasing. It follows that the Vaidya spacetime describes a radiative white hole that loses mass and becomes more and more electrically charged. We aim to extend the results obtained in the Vaidya spacetime to this geometry. If we consider the static solution (i.e. the Reissner-Nordström) situation, there are two horizons situation located at $r_\pm = m \pm \sqrt{m^2 - q^2}$. We prove that these two horizon are decreasing with u in the Vaidya-Bonnor spacetime and that r_- is going to zero for a finite time. We are also studying the behaviour of the other integral lines of the incoming main direction, and we want to focus on the situation of complete evaporation of the white hole.

4. **Dirac equation on curved spacetimes** We are working with Nabile Boussaïd on the Dirac equation on curved spacetime. The aim is to obtain estimates for the Dirac equation on curved spacetime. Technically, we want to obtain Morawetz estimates for the Dirac equation using the multiplier method without passing through the Klein-Gordon equation. This work is in progress.

5. **Maxwell-Klein-Gordon equations in Minkowski's spacetime [5]**. We study, with Grigalius Taujanskas, the behaviour of solutions to Maxwell's equations coupled to the Klein-Gordon equation in the massless case. The idea is to construct the conformal scattering for these fields on the Einstein cylinder. To do this we need to control the singularity that appears at spacelike infinity due to the lack of decay caused by the electric charge. We establish the existence of the conformally rescaled solution on the Einstein cylinder and the regularity of the field at the boundary of compactified Minkowski spacetime. The resolution of the Goursat problem at null infinity is equivalent to the construction of the scattering operator. The non trivial nature of the nonlinearity of the MKG system makes this resolution much more delicate than in the cases we have studied sofar. This work is in progress.

Research projects

1. **Study of the past horizon on the Robinson-Trautman spacetime**. As explained before, the method developed in the Vaidya spacetime to study the past horizon fails to yield results for the past horizon in the Robinson-Trautman solutions, as the incoming principal null directions (PND) are not geodetic anymore. Consequently, it is not sufficient to study solutions of the ordinary differential equation that governs the incoming null curves, i.e., the integral lines of the incoming PND. Therefore, we aim to investigate solutions to the second ordinary differential

geodesic equation.
By setting the initial data to be the past horizon in the Vaidya spacetime for $u \leq u_-$, we aim to obtain information about the behavior of the white hole horizon. This work will be separated into two steps. Firstly, we will consider an axisymmetrical metric, depending then only on u and θ. Then, we hope to extend our results to the pure radiative type D Robinson-Trautman metric studied in [4], i.e. metrics depending on u, θ and φ.

2. **Physical aspect of conformal scattering**. Conformal scattering is a powerful theory to study the propagation of a field on a given background, in particular in the case of dynamical spacetimes. This is because the theory extracts the information directly from the asymptotics of the geometry rather that through spectral analysis of the hamiltonian operator. Conformal scattering would gain a lot of physical meaning if one could obtain some understanding of the spacetime geometry from the scattering operator. The question is whether one can develop some inverse scattering theory from the conformal scattering construction. A first steps in this project would be to reconstruct the mass function of the Vaidya spacetime from the conformal scattering operator.

3. **Conformal scattering in a Vaidya spacetime with a mass varying on** $\mathbb{R}$. During my PhD I constructed a conformal scattering operator for the scalar waves on a dynamical spacetime for which the mass is constant near time-like infinities (i.e. the solution is the Schwarzschild solution near time-like infinities). On a compact time interval, however, the mass is decreasing and the metric is the Vaidya metric. As explained earlier, this choice was constrained by the fact that there are no decay laws for the scalar waves near time-like infinities in the Vaidya spacetime. It will be interesting to construct a conformal scattering operator for the scalar waves in a Vaidya spacetime for which the mass $m(u)$ varies for all $u \in \mathbb{R}$. The first step is to obtain a decay law for the scalar wave equation near the time-like infinities, as obtained by Dafermos and Rodnianski (2008). This could be done using the microlocal analysis tools. The second step is to obtain energy estimates in the spacetime for the scalar waves between the hypersurface of the initial data and the boundary of the conformal spacetime. We then define a trace operator that is one-to-one with closed ranged. Finally, using Hörmander methods, we solve a Goursat problem and prove that the image of the future trace operator is dense. This implies that this operator is an isomorphism and this allows us to define a conformal scattering operator which characterises the whole evolution of the scalar waves from their asymptotic behaviour (i.e. the trace of the field on the future/past boundary).

4. **Conformal scattering for Maxwell fields in Schwarzschild's and Vaidya's spacetime**. This research project will be conducted jointly with Yannick Herfray Associate Professor at the Denis Poisson Institute in Tours. This project focuses on the asymptotic analysis of systems of equations in mathematical physics admitting a gauge freedom, using conformal geometry methods. The scattering approach is considered as a possible answer to developing a holographic theory for this type of spacetime, aligning with recent work of Herfray on celestial holography (see, for example, [5]). The project can be organised as follows : The first equation we study is the Maxwell equation on black hole spacetimes. The goal is to construct a conformal scattering theory in Schwarzschild and Vaidya metrics. Two approaches are considered.
In the first approach, scattering is directly constructed for the electromagnetic field. The energy

on Cauchy hypersurfaces is naturally the L^2 norm of the field. It is conserved in Schwarzschild due to the presence of a temporal Killing field outside the black hole. In Vaidya, it is no longer conserved during radiative phases of spacetime, but it can be controlled using a priori estimates. The field is described in spinor form using a separation of self-dual and anti-self-dual parts, via the Dirac equation acting on a symmetric spinor with two indices.

The construction of the scattering theory proceeds as it was done for the scalar waves in the Vaidya spacetime [1]. We begin by establishing the existence of trace operators that associate the conformal restriction of the field to isotropic infinities in the future and past to the Cauchy data of the field. These operators are also initially constructed for smooth data with compact support. We then establish energy estimates in both directions between null infinity and the Cauchy surface. This requires to show that energy cannot accumulate near timelike infinity. To achieve this, we use results on the decay of Maxwell fields in the Schwarzschild metric by W. Inglese and F. Nicolò (2000) and P. Blue (2008). Trace operators are thus extended by density to all data with finite energy (without support constraints). The final step is to show that these operators are isomorphisms (isometries in the case of Schwarzschild) and establish that their range is dense (energy estimates ensure injectivity and closed range), which can be demonstrated by solving the characteristic Cauchy problem at isotropic infinity for smooth data with compact support. The article by J.-P. Nicolas (2016) on conformal scattering for waves in the Schwarzschild metric, as well as my work on the Vaidya metric (2023) and (2021), will be important tools.

The second approach is to develop the scattering theory at the vector potential level for the Maxwell field. Its interest is both conceptual, as it is a construction in terms of connection rather than curvature, and practical, providing essential results and methods for the third part of this project. A recent article by J.-P. Nicolas and G. Taujanskas (2022) develops this type of construction on asymptotically simple spacetimes. The situations in which we work (Schwarzschild and Vaidya metrics) have more symmetry, and we will start by exploring specific gauge choices that this type of geometry allows and the consequences in terms of scattering construction. The geometry of isotropic infinity and the associated asymptotic symmetries (BMS group) play a fundamental role in the potential-based construction.

45 Rue des Justices – 25000 – Besançon, France
06 86 26 78 00 • ✉ armand.coudray@univ-fcomte.fr
https://acoudray.perso.math.cnrs.fr/